Prof. Dr. E. Kostina
M. Sc. H. Stibbe

Ruprecht-Karls-Universität
Institut für Angewandte Mathematik

Numerical Optimization with Differential Equations 1 - WS 2018/2019

Exercise 4

Exercise 1

Let measurement data η_i be given with corresponding model responses $h_i(t_i, x(t_i), p)$ and measurement errors ϵ_i, $i = 1, \dots, M$. The measurements shall be independent.
We consider least-squares functionals as maximum-likelihood estimators.
Which assumptions on the distribution of the measurement errors have been made for the following functionals?

a. $\min \sum_{i=1}^{M} (\eta_i - h_i(t_i, x(t_i), p))^2$,

b. $\min \sum_{i=1}^{M} \frac{(\eta_i - h_i(t_i, x(t_i), p))^2}{\sigma_i^2}$ with $\sigma_i > 0$, $i = 1, \dots, M$,

c. $\min \sum_{i=1}^{M} w_i \cdot \frac{(\eta_i - h_i(t_i, x(t_i), p))^2}{\sigma_i^2}$ with $\sigma_i > 0$ and $w_i \in \{0, 1\}$, $i = 1, \dots, M$,

(6 Points)

Exercise 2

Let $J \in \mathbb{R}^{m \times n}$ be a matrix. Show that the *Moore-Penrose Pseudo Inverse* $J^\dagger$ exists and is uniquely determined by the four axioms

$$J^\dagger J J^\dagger = J^\dagger, \qquad J J^\dagger J = J, \qquad J^\dagger J = (J^\dagger J)^T, \qquad J J^\dagger = (J J^\dagger)^T.$$

(4 Points)

Exercise 3

Let $J_1 \in \mathbb{R}^{m_1 \times n}$ and $J_2 \in \mathbb{R}^{m_2 \times n}$ be matrices satisfying

- **(CQ)** $\text{Rang}(J_2) = m_2 \leq n$,
- **(PD)** $\text{Rang}\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = n \leq m_1 + m_2$.

Show

a. The matrix $J_1^T J_1 \in \mathbb{R}^{n \times n}$ is positive definite on the nullspace of J_2, i.e. $x^T J_1^T J_1 x > 0$ for all $x \in \mathbb{R}^n \setminus \{0\}$ with $J_2 x = 0$.

b. The matrix $\begin{pmatrix} J_1^T J_1 & J_2^T \\ J_2 & 0 \end{pmatrix}$ is regular.

(4 Points)

*Hand in solutions on **Tuesday**, November 20th, **at the beginning** of the lecture!*