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Numerical Optimization with Differential Equations 1 - WS 2018/2019

Exercise 7

Exercise 1

Consider for continuously differentiable $F_1 : \mathbb{R}^n \to \mathbb{R}^{m_1}$ and $F_2 : \mathbb{R}^n \to \mathbb{R}^{m_2}$ the constrained nonlinear least-squares problem

$$\begin{aligned} \min_x \quad & \tfrac{1}{2}\|F_1(x)\|_2^2 \\ \text{s.t.} \quad & F_2(x) = 0. \end{aligned}$$

Assume that $x^* \in \mathbb{R}^n$ satisfies $F_1(x^*) = 0$ and $F_2(x^*) = 0$ such that [CQ] and [PD] hold in x^*. Show that for constrained Gauß-Newton and arbitrary $\varepsilon > 0$ there exists a neighbourhood of x^* such that the assumptions of the local contraction theorem are satisfied in this neighbourhood with $\kappa \leq \varepsilon$.

(4 Points)

Exercise 2

Consider for continuously differentiable $F_1 : \mathbb{R}^n \to \mathbb{R}^{m_1}$ and $F_2 : \mathbb{R}^n \to \mathbb{R}^{m_2}$ the constrained nonlinear least-squares problem

$$\begin{aligned} \min_x \quad & \tfrac{1}{2}\|F_1(x)\|_2^2 \\ \text{s.t.} \quad & F_2(x) = 0. \end{aligned}$$

Let $A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$ with $A_i \in \mathbb{R}^{m_i' \times (m_1+m_2)}$ such that x is a solution of the original problem if and only it is a solution of the following transformed problem:

$$\begin{aligned} \min_x \quad & \tfrac{1}{2}\|A_1 F(x)\|_2^2 \\ \text{s.t.} \quad & A_2 F(x) = 0. \end{aligned}$$

(a) Show affine invariance of the Gauß-Newton method: $x^k = y^k$ for all $k \geq 0$, where $(x^k)_k$ is the sequence of Gauß-Newton iterates applied to the original problem und $(y^k)_k$ the sequence of Gauß-Newton iterates applied to the transformed problem, with $y^0 := x^0$.

(b) Show that $A_1 = \begin{pmatrix} U & C_1 \\ 0 & C_2 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & C_3 \end{pmatrix}$ satisfies this assumption, where $U \in \mathbb{R}^{m_1 \times m_1}$ is orthogonal, $C_3 \in \mathbb{R}^{m_2 \times m_2}$ is non-singular and $C_1 \in \mathbb{R}^{m_1 \times m_2}$ and $C_2 \in \mathbb{R}^{(m_1'-m_1) \times m_2}$ are arbitrary.

(c) Show that x is a solution to the original problem if and only if x is a solution to the problem

$$\begin{aligned} \min_x \quad & \tfrac{1}{2}\|F_1(x)\|_2^2 + \tfrac{1}{2}\|F_2(x)\|_2^2 \\ \text{s.t.} \quad & F_2(x) = 0. \end{aligned}$$

(6 Points)

*Hand in solutions on **Tuesday**, December 11th, **at the beginning** of the lecture!*