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Numerical Optimization with Differential Equations 1 - WS 2018/2019

Exercise 8

Exercise M5

Given is the parameter estimation problem

$$\min_{x_1,x_2} \sum_{i=1}^{6} \Big(h(t_i, x_1, x_2) - y_i \Big)^2$$

with the measuring function

$$h(t, x_1, x_2) = x_1 \Big(1 - \exp(-t x_2) \Big).$$

We obtain the measuring data $Y = (y_1, \ldots, y_6) = (8.3, 10.3, 19.0, 16.0, 15.6, 19.5)$ from six experiments at the time moments $T = (t_1, \ldots, t_6) = (1, 2, 3, 4, 5, 7)$. We assume independent and normally distributed measuring errors with expected value 0 and variance 1.

Solve the following tasks with Matlab:

1. Use the generalized Gauss-Newton method to do a parameter estimation for identifying the unknown parameters (x_1, x_2). Use the inital values $(x_1, x_2) = (20, 0.3)$ for the iteration. The necessary number of Gauss-Newton iterations and the estimation of the parameters should be displayed using the Matlab command *display(...)*.

2. Calculate the linear approximation of the covariance matrix in the solution point x^* of the Gauss-Newton method and display it.

3. Calculate for every single parameter the "exact" confidence intervals using the covariance matrix and display it (use $\gamma^2 = 10.6$).

4. Plot the linear confidence region

   $$\{x | \|J(x^*)(x - x^*)\|_2^2 \leq \gamma^2\}$$

   and compare it to the nonlinear confidence region

   $$\{x | \|F(x)\|_2^2 - \|F(x^*)\|_2^2 \leq \gamma^2\}.$$

   Again, use $\gamma^2 = 10.6$.

(8 Points)

Submit your Matlab solutions until ***Tuesday****, January 8th,* ***11:00 AM*** *by email to your tutor!*