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Numerical Optimization with Differential Equations 1 - WS 2018/2019

Exercise 10

Exercise 1
In the lecture you got to know the Mayer and the Lagrange cost functional.

a) Show, that every Mayer cost functional can be transformed into a Lagrange cost functional.

b) Show, that this is also possible in reverse.

(4 Points)

*Hand in solutions on **Tuesday**, January 15th, **at the beginning** of the lecture!*

Exercise 2
We assume that we have a optimal control software, that solves problems of the following type

$$\begin{aligned} \min_{y,u,y_0} \quad & \phi(y(t_f)) \\ \text{s.t. } & \dot{y}(t) = f(y(t), u(t)), \ t \in [t_0, t_f] \\ & y(t_0) = y_0 \\ & u \in \Omega \end{aligned}$$

where Ω is a subset of the space of piecewise linear functions on a given grid. We have a problem of the following type

$$\begin{aligned} \min_{y,u,t_f,q} \quad & \int_{t_0}^{t_f} L(y(t), u(t), q)dt \\ \text{s.t. } & \dot{y}(t) = f(t, y(t), u(t), q), \ t \in [t_0, t_f] \\ & y(t_0) = y_0 \\ & u \in \Omega, \ q \in \Gamma \end{aligned}$$

where Ω is as above and $\Gamma \subseteq \mathbb{R}^n$. Transform this problem, such that it can be solved by the above software.

(4 Points)

Exercise 3
We consider the problem of time optimal driving which can be described by the following optimal control problem

$$\begin{aligned}
&\min_{x,u} \int_0^T 1dt \\
&\text{s.t. } \dot{x}_1(t) = x_2(t) \\
&\qquad \dot{x}_2(t) = u(t) \\
&\qquad x_1(0) = x_2(0) = 0 \\
&\qquad x_1(T) = 300,\ x_2(T) = 0 \\
&\qquad x_i(t) \geq 0\ \forall t \in [0, \tau],\ u(t) \in [-2, 1].
\end{aligned}$$

Solve this problem using the Maximum Principle.

(4 Points)

*Hand in solutions on **Tuesday**, January 22nd, **at the beginning** of the lecture!*